Analysis of noise-controlled shear-layers

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Despite very similar flow-structures, the uncontrolled and controlled shear-layers of Wei & Freund1 were found to generate radically different sound fields (a difference of up to 11dB). In this work we analyse these flows—with a view to better understanding what led to this sound reduction—using two approaches: (1) we analyse the in-flow pressure fields in the $\kappa_x - \omega$ domain, where we find that the acoustically-matched components of the controlled flow have been reduced substantially, while the global structure remains very similar; and, (2) we use causality-correlations to study the kernel of the Lighthill solution: the spatiotemporal source-correlation fields are found to have been dramatically degenerated by virtue of very subtle changes in the controlled flow’s evolution.

I. Introduction

A number of approaches for analysis of the noise-controlled shear-layers of Wei and Freund1 were developed and tested during the second European Forum on Flow Control (held at the Laboratoire d’Études Aérodynamiques, Poitiers in 2006). The difficulty of identifying the component of the source dynamic which was efficient in the excitation of progressive pressure fluctuations was a central issue, and constituted the focus of the analysis methodologies applied. Two basic analysis strategies are pursued in this work. The first involves performing two-dimensional Fourier transforms of the hydrodynamic pressure fields of the controlled and un-controlled mixing-layers, from $p(x, y, t)$ to $\hat{p}(\kappa_x, y, \omega)$, in order to better understand changes in the spectral make-up of the ‘radiating’ components by means of the dispersion relation $\omega = \kappa c$. Colonius, Lele, and Moin13 give another form of the radiation criterion by performing a Fourier transform only in the streamwise direction and time and considering the radiation criterion $\omega > \kappa_x c$. A similar radiation criterion is used by Freund14. Only source components whose space and time scales satisfy this criterion will be efficient in coupling with the farfield. These components are

II. Wavenumber-frequency spectra

Ffowcs-Williams11 and Crighton12 demonstrate that if a Lighthill-like analogy is used to define a source quantity, the differential equation which describes the sound production problem leads to a radiation criterion in the form of the dispersion relation $\omega = \kappa c$. Colonius, Lele, and Moin13 give another form of the radiation criterion by performing a Fourier transform only in the streamwise direction and time and considering the radiation criterion $\omega > \kappa_x c$. A similar radiation criterion is used by Freund14. Only source components whose space and time scales satisfy this criterion will be efficient in coupling with the farfield. These components are

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referred to as *acoustically matched*. One of the major difficulties which arises when source terms are analysed in subsonic turbulence is that the majority of the source dynamic does not satisfy this criterion: most of the source structure is *not* acoustically matched. The noise-controlled shear-layers of Wei and Freund\( ^{1} \) provide an excellent example of this difficulty: despite almost identical flow dynamics, the controlled shear-layers produce radically different sound fields.

We perform two-dimensional Fourier transforms on the pressure fields extracted from the ‘noisy’ and ‘quiet’ flows, from \( p(x, y, t) \) to \( \hat{p}(k_x, y, \omega) \) in order to study the source structure in acoustically-matched regions of \( k - \omega \) space. This is based on the fact that the pressure fluctuation is closely related to the sound production. The transform in discretised form is defined by

\[
\hat{p}(k_x, y, \omega) = \frac{1}{MN} \sum_{m} \sum_{n} p(x_m, y, t_n) e^{-i2\pi k_x x_m - i2\pi \omega t_n}. \tag{1}
\]

This permits the subtle differences between the ‘noisy’ and ‘quiet’ flow structures to be more clearly understood. A typical result is shown in figures 1, where the distribution of spectral energy, \( \hat{p}\hat{p}^* \) in the phase plane \((k_x, \omega)\) is plotted.

\( \text{Figure 1. The energy spectrum } \hat{p}\hat{p}^* \text{ at } y = 0: \text{ (a) } & \text{ (c) “noisy” flow; (b) } \text{ & (d) “quiet” flow.} \)
two wedge-shaped areas above and below between Mach 1.2 and Mach -0.8 lines. A zoom of these areas is shown in figure 1 ((c) & (d)). This shows the significant suppression of radiation capable modes in the “quiet” flow.

III. A Lighthill-based analysis

In this section we will present an analysis strategy which is rooted in the Lighthill framework. We use causality correlations to study the sound producing mechanisms present in the controlled and uncontrolled shear-layers.

In anticipation of criticisms which will no-doubt arise regarding our choice of the Lighthill framework—due to the fact that the Lighthill source term is known to comprise many physical phenomena which do not correspond to sound ‘production’ per se—we can make the following comments.

In so far as the aeroacoustic problem can be solved once the Lighthill source term is known in its entirety, the latter constitutes a truly interesting quantity to study. The reason for this is that once we dispose of a means of accessing the full source term (via a numerical simulation for example), we can study the space-time structure of a quantity which we know to give the acoustic farfield when convolved with a free space Green’s function; the fact that it contains flow-acoustic interactions and other ‘un-source-like’ physical phenomena within the rotational region of the flow is in some respects irrelevant; it is perfectly justified to consider the jet to comprise an extremely complex ensemble of physical processes, which are very nicely, and compactly, represented by the Lighthill source term, and which together drive progressive pressure modes in the farfield.

With this in mind, an interesting first question is: how are these progressive modes excited? The Lighthill formulation provides a relatively straightforward theoretical framework for addressing this question.

III.A. Revisiting Lighthill’s formulation

For an isothermal flow the Lighthill wave equation can be written as:

\[ \Box^2 p(x, t) = \frac{\partial^2}{\partial x_i \partial x_j} (\rho(x, t) u_i(x, t) u_j(x, t)) = q(x, t). \]

(2)

This describes a homogeneous medium at rest which is driven by a space-time structure given by the right-hand-side. For the moment let’s treat it as just that—without getting into complex arguments as to what kind of physical phenomena this space-time structure may involve: it can be simply and correctly considered to comprise the entire compressible, non-linear dynamic of the free-shear flow.

In order to understand how the said space-time pattern generates progressive pressure modes capable of attaining the farfield, we can look to the solution to the problem:

\[ p(x, t) = \frac{1}{4\pi} \int_{V} \frac{q(y, t - \frac{|x - y|}{c})}{|x - y|} d^3 y. \]

(4)

This equation translates the linear, compressible response of a uniform, homogeneous base flow, \( p(x, t) \), to the space-time structure \( q(y, t) \), and what the equation tells us is that if we consider the excitation field in a distorted spatiotemporal reference frame, \( q(y, t - \frac{|x - y|}{c}) \), the farfield pressure is given by simply summing all the points of that distorted field. We reiterate these very basic facts as they form the basis of the analysis used here. The instantaneous acoustic intensity radiated from the system—a measure of the sonic energy generated by the flow—can be written as

\[ p(x, t)p(x, t) = \frac{1}{16\pi^2} \int \int \frac{q(y, t - \frac{|x - y|}{c})}{|x - y|} \frac{q(y', t - \frac{|x - y'|}{c})}{|x - y'|} d^3 y d^3 y'. \]

(5)

\[ \text{by Lighthill source term we mean } \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j}, \text{ and not simply } T_{ij} = \rho u_i u_j \]
The right-hand side of this equation provides a means of localising acoustically efficient regions of the flow. For a farfield observer at $x_o$, the acoustic intensity radiated by the flow is

$$p(x_o, t)p(x_o, t) = \frac{1}{16\pi^2} \int \int \frac{q(y, t - \frac{|x_o - y|}{c})}{|x_o - y|} \frac{q(y', t - \frac{|x_o - y'|}{c})}{|x_o - y'|} d^3y d^3y'. \quad (6)$$

The local contribution from $y_o$ to the instantaneous farfield intensity at $x_o$ is given by

$$p(x_o|y_o, t)p(x_o|y_o, t) = \frac{1}{16\pi^2} \int \int \frac{q(y_o, t - \frac{|x_o - y_o|}{c})}{|x_o - y_o|} \frac{q(y', t - \frac{|x_o - y'|}{c})}{|x_o - y'|} d^3y'. \quad (7)$$

The integrand on the right-hand-side describes a retarded-time—or spatiotemporally-distorted—correlation between the instantaneous source level $q(y_o, t - \frac{|x_o - y_o|}{c})$ and the value of $q$ at all other points in a similarly distorted reference frame, i.e. $q(y', t - \frac{|x_o - y'|}{c})$; the sum over this correlation-field gives the instantaneous contribution from the point $y_o$, and so we see the extent to which the acoustic efficiency at a point in the source field is governed by its relation to its neighbouring points (in a distorted space-time).

When considered in the reference frame $(y, t - \frac{|x_o - y|}{c})$, we thus see that it can be relatively clear what noisy or silent flow structures will look like; the challenge is to be able to relate the structure as seen in the distorted reference frame to what is happening in undistorted, physical space-time $(y, t)$, and, finally, to be able to relate all this to more intuitively amenable flow phenomena. We consider the temporal evolution of the pressure and the pressure intensity at a single farfield observer point, $x_o$, and we simultaneously study the temporal evolution of both the integrands of equations 6 and 7, and the integration operations, time-step by time-step.

### III.B. Causality correlations

An alternative means of studying the kernel of the Lighthill solution is possible as equations 6 and 7 can be rewritten with the farfield pressure replacing $q(y, t - \frac{|x_o - y|}{c})$ and $q(y_o, t - \frac{|x_o - y_o|}{c})$ respectively:

$$p(x_o, t)p(x_o, t) = \frac{1}{16\pi^2} \int p(x_o, t) \frac{q(y', t - \frac{|x_o - y'|}{c})}{|x_o - y'|} d^3y', \quad (8)$$

$$p(x_o|y_o, t)p(x_o|y_o, t) = \frac{1}{16\pi^2} \int p(x_o, t) \frac{q(y_o, t - \frac{|x_o - y_o|}{c})}{|x_o - y_o|}. \quad (9)$$

This is the basis of the causality correlation approaches which were developed in the 1970s and 1980s, and applied experimentally. The instantaneous retarded-time correlation between the farfield pressure and the Lighthill source term is formally equal to the solution to the aeroacoustic problem. In this it is more than a simple test for causality, it provides a less complicated means of evaluating the details of the retarded-time source-source correlations which are embodied by the integrands of equations 6 and 7. However, two important differences set the work described here apart from the aforesaid experimental work: (1) as the study is numerical, the source term can be considered in its entirety, whereas the experimental approaches in the 70s and 80s were restricted to incomplete source representations; (2) the study can avail of a full volumetric treatment of the problem, which allows the subtleties of the cancellation/interference mechanisms to be correctly and completely addressed. These two points constituted a major limitation of the experimental approach (one which prevented the scientists in the 1970s and 80s from drawing concrete quantitative conclusions from their analyses).
III.C. Some preliminary results

The results presented in what follows are for the uncontrolled and internal-energy-controlled 2D shear-layers: the reader can refer to Wei & Freund\(^1\) for more details regarding both the simulations and the flow characteristics. In a first instance the time-averaged local contribution from the source-system to the sound pressure intensity at the farfield point \(x_o\) can be obtained by means of the following equation:

\[
\begin{align*}
p(x_o|y_o,t)p(x_o|y_o,t) & = \frac{1}{16\pi^2} p(x_o,t) q(y_o,t - \frac{|x_o - y_o|}{c}) \frac{1}{|x_o - y_o|} \int q(y_o,t - \frac{|x_o - y_o|}{c}) q(y',t - \frac{|x_o - y'|}{c}) d^3 y'.
\end{align*}
\]

\((10)\)

\((11)\)

Figure 2. (a) Time-averaged, retarded time source-farfield correlation: a measure of the local contribution to the farfield intensity; (b) Top: Farfield pressure time history; middle: retarded-time source field corresponding to black dot on farfield time-trace; bottom: source-field in undistorted physical space-time—a single time delay has been applied such that the snapshot approximately corresponds to the black dot on the farfield pressure time-trace.

The result of this operation is shown in figure 2(a), where a series of positive and negative zones are observed: these correspond, respectively, to ‘sources’ and ‘sinks’ in terms of local, time-averaged, contributions to the farfield sound pressure intensity. This is the kind of result which was possible experimentally using simple two-point correlations. It can be an interesting way of focusing our attention on regions of the flow which made, on average, positive or negative contributions to the pressure intensity at a given farfield point. An advantage of the numerical approach is that we can ‘relive’ the full space-time structure, either in physical or distorted (retarded-time) space-time, but with our attention now turned to the said ‘sources’ and ‘sinks’: such a snapshot is shown in figure 2(b).

It is worth emphasising that the bottom sub-figure in 2(b)—as it is in the distorted, retarded-time spatiotemporal reference-frame—provides a highly visual means of understanding how the flow-structure led to the instantaneous farfield pressure amplitude, as it is simply the sum over the flow domain which gives the farfield amplitude: source-fields with a very regular, spatially oscillatory flow structure (in the retarded-time reference-frame) which persists in time, will lead to maximal cancellation and therefore low farfield levels; source-fields with less regular structures (in the retarded-time reference-frame) will result in a non-zero residuum after summation, and this will leave a net positive or negative pressure amplitude in the farfield. As mentioned earlier, the challenge is to relate these ‘noisy’ or ‘quiet’ flow-patterns to the physical space-time structures.
III.D. Instantaneous analysis

Let us now examine the instantaneous, retarded-time source-farfield correlations (equation 9), which corresponds to the source-source correlation comprised by the integrand in equation 7. We move the time delay from the source field to the pressure field, such that the source (colourmap in top subfigure), vorticity (contours in top sub-figure) and correlation fields (colourmap in middle sub-figure) can be observed in physical space-time. However, this means that a given snapshot no-longer corresponds to a single point in the farfield time-history (the black dots on the farfield signature show the limits); in order to understand the interference mechanism responsible for a single farfield timestep we must step through the temporal evolution of the correlation-map over a period of time long enough for all of the field points to contribute. The black contours in the middle sub-plot do identify partial integration paths: as we step through the movies, these arcs move downwards across the correlation field—which moves from left to right; by following one of these arcs as it travels across the flow domain, summing its integrals at each time-step, we obtain the appropriate interference pattern which led to the instantaneous farfield intensity: and it is this which gives insight as to why the flow produced a given farfield signature.

In figure 3 a variety of different kinds of flow structure are shown. When we study these, using the instantaneous, retarded-time correlation field, and the farfield pressure signal, we are led to a clearer understanding as to why each of the flow structures generated their corresponding farfield fluctuation. Figure 3(a) & (b) shows examples of highly irregular flow patterns, with high local levels of correlation. The cancellation mechanism is relatively ineffective (integration along a black arc as it moves through the correlation field), and the result is high positive and negative farfield pressure fluctuations—this is in fact one of the noisiest events in the time history shown.

Figure 3(c) shows a different kind of structure: relatively high correlation levels are observed (colourscale of middle plot), but the regularity of the structure as the black arcs move through the field is such that summation leads to low farfield levels. Figure 3(d) is different again: in this case the correlation levels are practically non-existent and so the source structure is entirely ineffective as a source. It can furthermore be seen that the flow/source structure (top sub-plot) comprises a very regular train of vortical structures.

The instantaneous correlations are thus seen to provide interesting insights into the character of different kinds of ‘noisy’ or ‘quiet’ flow structures; these insights depend both on the instantaneous correlation levels, and the global structure of the correlation field in the distorted spatiotemporal reference-frame.

III.E. Comparing noisy and quiet flows

Let us first compare time-averaged source maps, with and without control. This is shown in figure 4. It is important to note that the correlation levels have been normalised by the farfield pressure \( r_{ms} \), and so there is in fact nearly an order of magnitude difference between the sourcemaps. We observe that in terms of the time-average local source-contributions, there has been only slight ‘structural’ modification.

A further comparison between the noisy and quiet flows can be made by studying their evolution from time-step 465 to 582, during which both flow-fields generate a farfield pressure fluctuation comprising three periods. Again while the correlation maps sometimes look similar (see figure 5 for example), there is an order-of-magnitude difference in level: the principal effect of the control is manifest in this: the structure of the controlled flow is such that the retarded-time source correlation has been considerably degenerated, by very slight-but-appropriate modification of the space-time flow structure. What we see in these highly disparate correlation levels is the space-time equivalent of the spectral differences observed in the \( \omega > \kappa_{x_c} \) region of the \((\omega - \kappa_x)\) domain \( 1 \). Thus, while the vorticity/source snapshots look very similar, the subtle differences in their evolution is sufficient to cause the correlation-and-sum operation to be spectacularly less efficient in the controlled flow. On other occasions (figure 489), in addition to this difference in level there is also a

\[b\] Remember the colourmap amounts to the retarded-time correlation between each point in the domain and all other source points, integrated over the entire flow domain: it is therefore a measure of the real importance of the local source fluctuation with respect to its retarded-time neighbours: we are seeing a source-field which is weighted such that the most acoustically important events are highlighted, while the acoustically ineffective fluctuations are suppressed.
Figure 3. (Top of each subplot: $q(y, t)$; middle: $q(y, t)p(x_o, t + |x_o - y|)$ (colourmap) and $p(x_o, t + |x_o - y|)$ (black contours: dashed=negative, solid=positive); bottom: farfield pressure time-history (solid black line), and colours corresponding to $p(x_o, t + |x_o - y|)$ contours in middle plot. (a) snapshot 204; (b) snapshot 234: Examples of strong source-correlation levels and irregular structure: associated with a large farfield pressure fluctuation; (c) snapshot 267; example of strong correlation level, but regular structure leading to effective cancellation; (d) snapshot 282: example of weak source-correlation levels, and therefore low sound production: interference pattern irrelevant.
Figure 4. Time-averaged, retarded time source-farfield correlations: (a) Uncontrolled flow; (b) flow controlled by internal energy source

difference in the form of the correlation map. The evolution from snapshot 480 to 489 is accompanied by such a change: while the uncontrolled flow-structure’s evolution comprises a correlation-map which remains ‘coherent’ and qualitatively similar to the source-field, the correlation-map of the controlled flow tends to be less structured; when we study the source-fields we see that this de-structuring of the correlation-map is related to small-scale perturbations in the vorticity/source structure

IV. Conclusion

A number of analysis approaches have been applied to the noise-controlled two-dimensional shear-layers of Wei & Freund.\(^1\) Firstly, by means of Fourier transformation of the in-flow pressure field, from \(p(x, y, t)\) to \(\hat{p}(\kappa_x, y, \omega)\), it was shown that the energy of the ‘radiating’ components of the flow structure has been considerably reduced, while the global flow-structure remains relatively unchanged. A second methodology was used, based on causality correlations between the Lighthill source term and the acoustic pressure sampled at a single farfield point. From these correlations we see that, despite similar flow/source fields, the retarded-time correlation levels are very largely reduced in the controlled flow. This means that the space-time structure of the controlled flow has been adjusted, very slightly, but sufficiently for the correlation-and-sum operation manifest in the farfield solution to Lighthill’s equation to be rendered considerably less efficient: the two-point correlations have been strongly degenerated.

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Figure 5. (a) Snapshot 480; (b) snapshot 489. Uncontrolled flow on the left; controlled flow on the right. Correlation and pressure levels have been scaled by the farfield rms: the correlation colour-maps are thus not directly comparable: there is nearly an order of magnitude difference.

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