

Optimal Control for Spacecraft to Rendezvous with a Tumbling Satellite in a Close Range

Zhanhua Ma

Department of Mech. & Aero. Eng., Princeton University
Princeton, NJ 08544, USA
zma@princeton.edu

Ou Ma and Banavara N. Shashikanth

Department of Mech. Eng., New Mexico State University
Las Cruces, New Mexico, 88003, USA
oma@nmsu.edu, shashi@me.nmsu.edu

Abstract— One of the most challenging tasks for satellite on-orbit servicing is to rendezvous and capture a non-cooperative satellite such as a tumbling satellite. This paper presents an optimal control strategy for a servicing spacecraft to rendezvous (in close range) with a tumbling satellite. The strategy is to find an optimal trajectory which will guide the servicing spacecraft to approach the tumbling satellite such that the two vehicles will eventually have no relative rotation. Therefore, a subsequent docking or capture operation can be safely performed. Pontryagin's maximum principle is applied in generation of the optimal approaching trajectory and the corresponding set of control force/torque profiles. A planar satellite chasing problem is presented as a case study, in which together with the maximum principle, the Lie algebras associated with the system are used to examine the existence of singular extremals for optimal control. Optimal trajectories for minimum fuel consumption are numerically simulated.

I. INTRODUCTION

There has been an increasing interest in satellite on-orbit autonomous servicing in the space industry recently. JAXA recently completed a technology demonstration mission ETS-7 [1]. NASA just did an autonomous rendezvous mission through the DART mission, where the mission was not completed due to more than expected fuel usage during rendezvous maneuvering [2]. DARPA is currently developing a more advanced technology demonstration mission to be launched in 2006 through the Orbital Express Program [3-5]. Germany and Canada are also jointly developing a robotics-based satellite rescue mission called TECSAS which will be launched likely in four years from now [6-7]. In order to perform on-orbit service, the servicing spacecraft has to first rendezvous and capture the satellite to be serviced in orbit. In a general 'satellite capture problem', we suppose that there is a target satellite and a chaser satellite flying in space. The target satellite (target) is moving with spinning or tumbling motions in an orbit (see Fig. 1), while the task of the chaser satellite (chaser) is to rendezvous with the target in space in a desired way and finally capture it. All of the current and past on-orbit servicing missions focus only on the capture of a cooperative satellite which is supposed to move smoothly in its orbit without rapid attitude changing. In reality, a malfunctioning satellite may spin or tumble in orbit. Such a satellite is considered as a non-cooperative satellite. Capture of a non-cooperative satellite is a tremendous challenge. Very few research works on the problem of capturing a tumbling satellite have been done. Most of the proposed methods

require a manipulator onboard the chaser satellite (e.g. [8-9]). Even with a very capable manipulator, the chaser still has to align with the tumbling satellite before any subsequent robotic operations can proceed. Sakawa studied the problem of controlling a single freely flying object to fly from one position and orientation to another in an optimal manner [10]. Matsumoto, et al. studied fly-by and optimal orbits for maneuvering to a rotating satellite [11]. Nakasuka and Fujiwara proposed a method for matching angular velocities between the chaser and target by changing the target's moments of inertia [12]. Fitz-Coy and Liu proposed a two-phase navigation solution for rendezvous with a tumbling satellite in 2D space [13].

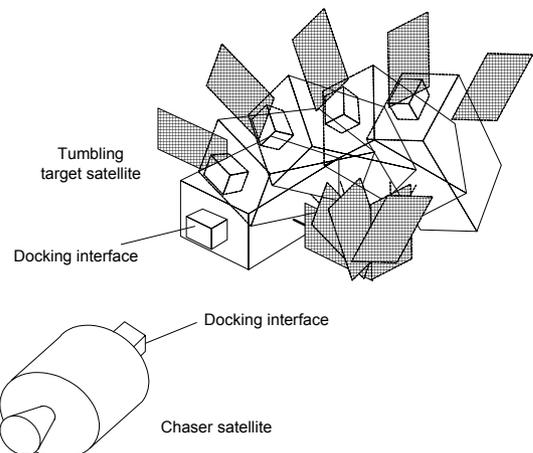


Fig. 1 A chaser spacecraft trying to capture a tumbling satellite

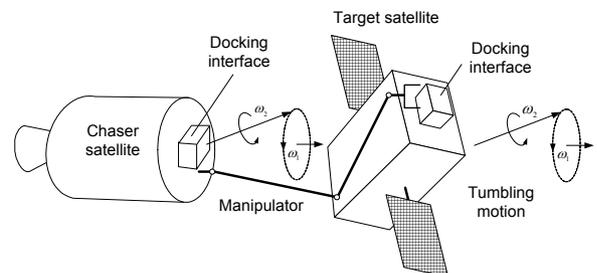


Fig. 2 The chaser spacecraft has captured the tumbling satellite

The research reported in this paper focuses on the close-range rendezvous problem for servicing satellite to capture a tumbling satellite with external forces/torques as control inputs. The two satellites are modeled as rigid bodies, with no constraints on the target's motion. Thus, for example, the target could be rapidly tumbling in space. The goal is to

control the motion of the chaser in an optimal manner so that it eventually flies at a close distance from the target without relative motion between the two (see Fig. 2). Without any relative rotation between the two vehicles, the subsequent capture operation becomes straightforward.

The control problem that we tackle is defined next. Supposing that the kinematics of the tumbling target satellite is perfectly known, design a control law, with optimality criteria, following which the chaser will move towards the target along the designed trajectory till there is no relative motion between the target and the chaser. In this control law design problem, no errors are considered and therefore it is a feed-forward control problem. The feedback control part as the second stage will be considered in future work. Two optimality criteria are considered: minimum time and minimum fuel-consumption. Optimization of fuel-consumption is gaining more attention by the space community after the DART mission [2]. In this paper both analytic and numerical approaches to the optimization problem are presented.

II. DYNAMICS MODELLING

A. Dynamics of the Two Satellites

The relative motion between the target and the chaser can be described in the target's body fixed frame. Further, the following assumptions will be made:

- The target's motion (position, orientation, and velocities) in space are known.
- Except for the control thrusts of the chaser, no other external forces are considered.

Assumption b) simplifies the analysis but will not affect the generality of developing the proposed control methodology. The possible external forces such as these due to the gravity gradient, Earth magnet, residual air drag, etc., are all much smaller than the thrust forces in the close-range rendezvous situation. With the above-listed two assumptions, it follows that the center of the mass of the target is moving along a straight line. The relative motion of the chaser with respect to the free-flying target is expressed in the target's body fixed frame by a system of first-order ordinary differential equations of the form:

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x}) \quad (1)$$

where $\mathbf{x}=[\mathbf{q},\dot{\mathbf{q}}]^T \in \mathbf{R}^{12}$, in which $\mathbf{q} \in \mathbf{R}^6$ is the vector of generalized coordinates representing the relative position and orientation of the chaser with respect to the target, the dot denotes differentiation with respect to time, and $\mathbf{f}(\mathbf{x}) \in \mathbf{R}^{12}$.

B. Affine Control System

Dynamic system represented by eq.(1) has no control inputs. It is now assumed that control forces and torques are applied to the chaser through thrusters or other means and that these forces and torques actuate all six degrees of freedom of the

chaser. With the inclusion of these control forces and torques, the system (1) gets replaced by the following affine control system expressed in the target's body fixed frame as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \quad (2)$$

where $\mathbf{G}(\mathbf{x}) = [\mathbf{0}, \mathbf{L}(\mathbf{x})]^T$ is a 12×6 matrix, $\mathbf{L}(\mathbf{x})$ is a 6×6 matrix and $\mathbf{u} \in \mathbf{R}^6$ is the vector of control inputs which is assumed to be bounded ($u_{i\min} \leq u_i \leq u_{i\max}$, $i = 1, \dots, 6$). The initial state of the system is assumed to be $\mathbf{x}(0) = \mathbf{x}_0$.

C. Control Objective

System (2) will be the basic model for studying the rendezvous problem. For this system, the objective of rendezvousing is defined as the control objective of *achieving zero relative motion* between the target and the chaser and then *maintaining such a state of zero relative motion* for the subsequent capturing process. More precisely, the control objective may be stated as: for some $\mathbf{c} = [\mathbf{c}_1, \mathbf{0}]^T \in \mathbf{R}^{12}$, where $\mathbf{c}_1 \in \mathbf{R}^6$, and $t_d > 0$,

$$\mathbf{x}(t) = \mathbf{c}, \quad t \in [t_d, \infty)$$

Here, t_d is the rendezvous time and \mathbf{c} is the constant position vector of the chaser in the target's body-fixed frame signifying zero relative motion between the two satellites.

To realize this control objective, we divide the problem into two stages. In the first stage, $t \in [0, t_d]$ and we seek a controller $\mathbf{u}^*(t)$ which will move the system from the initial point $\mathbf{x} = \mathbf{x}_0$ to the final point $\mathbf{x} = \mathbf{c}$ in phase space in an optimal manner subject to the bounded constraints on \mathbf{u} . The method of obtaining $\mathbf{u}^*(t)$, by applying optimal control theory, will be discussed in the next section.

Throughout the second stage, $t \in [t_d, \infty)$, $\mathbf{x} = \mathbf{c}$. Therefore, from (2), the controller $\mathbf{u}^{**}(t)$ for the second stage satisfies

$$\mathbf{0} = \mathbf{f}(\mathbf{c}) + \mathbf{G}(\mathbf{c})\mathbf{u}^{**}(t), \quad (3)$$

and these equations can be inverted to solve for $\mathbf{u}^{**}(t)$. Thus, the feed-forward controller $\mathbf{u}(t)$ for this problem can be obtained as: $\mathbf{u}(t) = \mathbf{u}^*(t)$, $t \in [0, t_d]$ and $\mathbf{u}(t) = \mathbf{u}^{**}(t)$, $t \in [t_d, \infty)$. The key step in designing this control law is to obtain $\mathbf{u}^*(t)$, which will be discussed in the next section.

III. OPTIMAL CONTROL

Different optimal controls can be designed for different optimality criteria, such as 'time optimal', 'fuel-consumption optimal', etc. In this section, we discuss time-optimal control. Achieving rendezvous between the two satellites in the shortest time (i.e. a minimal t_d) is very meaningful and appropriate for a docking mission in space. In order to find $\mathbf{u}^*(t)$, $t \in [0, t_d]$, techniques in time-optimal control theory

will be applied, in particular the Maximum Principle of Pontryagin [14].

Pontryagin's Maximum Principle provides necessary conditions for a controller to optimize a given cost functional [14]. The principle does not provide sufficient conditions in general and thus satisfying the necessary conditions of the principle provides only "good candidates" for the optimal controller. The choice of the "best candidate" will have to be made based on further analysis, typically numerical, or by exploiting particular features of the problem.

In a time-optimal problem, such as system (2), the cost functional is $J = \int_0^{t_d} f_0 dt$, where $f_0 = 1$, and J , or equivalently t_d , has to be minimized for the system evolution from a given initial condition $\mathbf{x}(0) = \mathbf{x}_0$ to a given final condition $\mathbf{x}(t_d) = \mathbf{c}$. The Maximum Principle states that a necessary condition for an optimal controller $\mathbf{u}^*(t)$ is that it maximizes the Pontryagin Hamiltonian

$$H(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{u}) = \boldsymbol{\lambda}^T \mathbf{f}(\mathbf{x}) + \sum_{i=1}^6 \boldsymbol{\lambda}^T \mathbf{g}_i(\mathbf{x}) u_i + \lambda_0 f_0 \quad (4)$$

with respect to \mathbf{u} , during $t \in [0, t_d]$. In other words, $\frac{\partial H}{\partial \mathbf{u}} = 0$ for $\mathbf{u} = \mathbf{u}^*$ in this time interval. Moreover, H should also be a constant during this time interval. In (4), $\boldsymbol{\lambda} \in \mathbf{R}^{12}$ is called the adjoint vector, λ_0 is a negative constant, $\mathbf{g}_i(\mathbf{x})$ corresponds to the i -th column vector of the matrix $\mathbf{G}(\mathbf{x})$, and u_i corresponds to the i -th component of \mathbf{u} . The components of the adjoint vector $\boldsymbol{\lambda}$ and the state variable vector \mathbf{x} form a canonical Hamiltonian system:

$$\dot{x}_i = \frac{\partial H}{\partial \lambda_i}, \quad \dot{\lambda}_i = -\frac{\partial H}{\partial x_i}, \quad \text{for } i=1, \dots, 12 \quad (5)$$

where the latter set of equations is called adjoint equations.

The optimal controller is obtained as $\mathbf{u}^*(t) = \mathbf{u}^*(\mathbf{x}(t), \boldsymbol{\lambda}(t))$. Substituting this in (5) and solving for $\mathbf{x}(t)$ and $\boldsymbol{\lambda}(t)$ then gives us, in principle, the optimal trajectory of the system.

Define a triple $(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{u})$ as an extremal if it satisfies the maximum principle. The objective now is to find these extremals. However, obtaining them from the necessary condition on the Pontryagin Hamiltonian alone is not straightforward. Indeed for affine control systems it is obvious that (4) can be maximized by boundary values of \mathbf{u} . This leads to the idea of a "bang-bang" control in which the optimal controller is piecewise constant, i.e. u_i will be either $u_{i\min}$ or $u_{i\max}$, during $t \in [0, t_d]$. Even without the complication of bang-bang controllers, finding solution for controllers satisfying the Maximum Principle can be quite a challenge due to the nonlinearity and high dimensions of the system. Further, finding extremals can sometimes be

hampered due to lack of initial conditions of $\boldsymbol{\lambda}$.

Thus, besides analyzing the optimal control problem by using theoretical approaches, such as theorems due to Sontag and Sussmann [15] for examining the existence of singular extremals, we also consider numerical simulation methods since these can provide more direct information, such as generating actual trajectories. Sakawa-Shindo's (SS) algorithm [10, 16], because of its Maximum Principle based nature, simple structure, and good convergence performance, has been chosen for numerical simulations in this paper. In the next section, a planar example of the rendezvous problem will be discussed in which both the theoretical and numerical methods due to the above authors will be applied.

IV. EXAMPLE

D. Dynamic Model of the Relative Motion Between Target and Chaser

In the planar example shown in Fig. 3, the mass center of the target O_1 is assumed to be moving along a straight line at a constant speed. A translating reference frame XY is fixed to point O_1 . Consider also a body-fixed frame X_1Y_1 , attached to the target also at O_1 . The two frames are assumed to be initially coincident. The target with its body fixed frame X_1Y_1 is rotating at a constant angular velocity Ω about the axis through O_1 perpendicular to the plane. A body fixed frame X_2Y_2 is attached to the chaser at its mass center O_2 , whose coordinates in the XY frame are (x, y) . The orientation of the chaser is denoted by θ , which is defined as the angle between the X_2 and X axes. There are two external forces u_1 and u_2 , respectively in X_2 and Y_2 directions, and one external torque u_3 , in the direction perpendicular to the plane, working as control inputs to the chaser. The motion of the chaser with respect to the XY frame can then be described as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ \omega \\ (u_1 \cos \theta - u_2 \sin \theta) / m \\ (u_1 \sin \theta + u_2 \cos \theta) / m \\ u_3 / I_z \end{bmatrix} \quad (6)$$

where m is the mass of the chaser, I_z is the polar moment of inertia of the chaser about the point O_2 , (x, y) are the coordinates of O_2 in the XY frame, dot means time derivative and v_x , v_y and ω represent the translational and rotating velocities of the chaser observed in the XY frame.

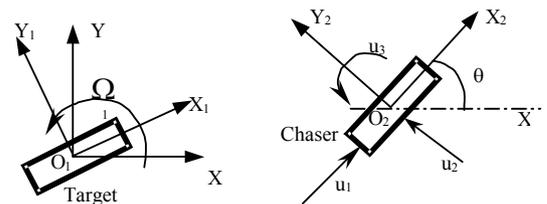


Fig. 3 Chaser and Target's motions in a plane

The position vector with respect to the XY frame, $[x \ y]^T$, can be expressed in the X_1Y_1 frame as $[x_r \ y_r]^T$ by using the orthogonal transformation $[x_r \ y_r]^T = \mathbf{A}[x \ y]^T$ where

$$\mathbf{A} = \begin{bmatrix} \cos \Omega t & \sin \Omega t \\ -\sin \Omega t & \cos \Omega t \end{bmatrix}$$

in which t represents time. System (6) can then be rewritten in the target's body-fixed frame X_1Y_1 in the form of an affine control system:

$$\dot{\mathbf{x}}_r = \mathbf{f}(\mathbf{x}_r) + \mathbf{G}(\mathbf{x}_r)\mathbf{u} \quad (7)$$

where subscript ' r ' indicates the relative motion of the chaser with respect to the target, observed from the target's body-fixed frame X_1Y_1 . The state-variable vector in (7) is defined as $\mathbf{x}_r = [x_r \ y_r \ z_r \ v_{xr} \ v_{yr} \ \omega_r]^T$, the \mathbf{f} and \mathbf{G} are

$$\mathbf{f}(\mathbf{x}_r) = \begin{bmatrix} v_{xr} \\ v_{yr} \\ \omega_r \\ \Omega^2 x_r + 2\Omega v_{yr} \\ \Omega^2 y_r - 2\Omega v_{xr} \\ 0 \end{bmatrix}$$

and

$$\mathbf{G}(\mathbf{x}_r) = [\mathbf{g}_1 \ \mathbf{g}_2 \ \mathbf{g}_3] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \cos \theta_r & -\sin \theta_r & 0 \\ \sin \theta_r & \cos \theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and the normalized control input vector \mathbf{u} is defined in the following way: If the physical control input is $\hat{\mathbf{u}} = [\hat{u}_1 \ \hat{u}_2 \ \hat{u}_3]^T$, then

$$\mathbf{u} = [u_1 \ u_2 \ u_3]^T = [\hat{u}_1 / m \ \hat{u}_2 / m \ \hat{u}_3 / I]^T.$$

With such a definition the theoretical design and analysis of the control system can be performed without considering specific mass and inertia distribution of the staellite.

Note that $\theta_r = \theta - \Omega t$ and $\omega_r = \omega - \Omega$. As defined in Section III, the control objective is to transfer the state variable \mathbf{x}_r from its initial state (when $t=0$)

$$\mathbf{x}_{r0} = [x_{r0} \ y_{r0} \ z_{r0} \ v_{xr0} \ v_{yr0} \ \omega_{r0}]^T$$

to a final state $\mathbf{x}_{rf} = [x_{rf} \ y_{rf} \ z_{rf} \ 0 \ 0 \ 0]^T$ in minimum time with the control inputs bounded ($u_{i\min} \leq u_i(t) \leq u_{i\max}$, $i=1, \dots, 3$).

E. Analysis of the Time-Optimal Control Problem

By Pontryagin's Maximum Principle applied to the planar

example, the Hamiltonian needed to be maximized during the whole time interval is

$$H(\mathbf{x}_r, \boldsymbol{\lambda}, \mathbf{u}) = \boldsymbol{\lambda}^T \mathbf{f}(\mathbf{x}_r) + \sum_{i=1}^3 \boldsymbol{\lambda}_i^T \mathbf{g}_i(\mathbf{x}_r) u_i + \lambda_0 f_0 \quad (8)$$

where $f_0=1$ and λ_0 is a negative constant. The corresponding adjoint equations are

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \\ \dot{\lambda}_4 \\ \dot{\lambda}_5 \\ \dot{\lambda}_6 \end{bmatrix} = \begin{bmatrix} -\lambda_4 \Omega^2 \\ -\lambda_5 \Omega^2 \\ a \\ -\lambda_1 + 2\lambda_5 \Omega \\ -\lambda_2 - 2\lambda_4 \Omega \\ -\lambda_3 \end{bmatrix} \quad (9)$$

where

$$a = \frac{1}{m} [\lambda_4 (u_1 \sin \theta_r + u_2 \cos \theta_r) - \lambda_5 (u_1 \cos \theta_r - u_2 \sin \theta_r)].$$

In an optimal control problem, if there exists a nonempty time interval such that the switching function $\phi_i(t) = \boldsymbol{\lambda}(t)^T \mathbf{g}_i(\mathbf{x}(t))$, is identically zero in it, the corresponding extremal $(\mathbf{x}_r, \boldsymbol{\lambda}, \mathbf{u})$ is called u_i -singular. It is totally singular if it is u_i -singular for all I [17]. For system (7) of the planar example, computation of the following Lie brackets shows that

1) $[\mathbf{g}_i, \mathbf{g}_j]=0$ for $i, j \in \{1, 2, 3\}$;

2) $\{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3, [\mathbf{f}, \mathbf{g}_1], [\mathbf{f}, \mathbf{g}_2], [\mathbf{f}, \mathbf{g}_3]\}$ are linearly independent.

Thus, by the theorems due to Sontag and Sussmann [15] (Lemma 3.1 and Corollary 3.2) for time-optimal control, it is concluded that there is no totally singular extremal. That is, if an extremal is u_i, u_j -singular for $i, j \in \{1, 2, 3\}$, then it is not u_k -singular for $k \neq i, j$ and thus u_k must be bang-bang. This result implies that in a time-optimal control candidate $\mathbf{u}^* = [u_1 \ u_2 \ u_3]^T$ at least one element changes in a bang-bang manner with time. In the next subsection we will see the numerical counterpart of this conclusion.

C. Numerical Simulation and Results Analysis

The SS algorithm [10, 16] was developed for searching continuous optimal-control candidates satisfying Pontryagin's Maximum Principle. In particular, it can solve the fuel consumption optimal control problem with specified initial and final conditions in a fixed time interval [10]. Firstly, a fixed initial time t_1 is chosen. For the system (7), in the time interval $[0, t_1]$, a fuel consumption optimal controller should minimize the cost functional

$$J(\mathbf{u}) = \frac{1}{2} \sigma \|\mathbf{x}_r(t_1) - \mathbf{x}_{rf}\|^2 + \gamma \int_0^{t_1} (|u_1(t)| + |u_2(t)| + |u_3(t)| / R) dt \quad (10)$$

where the first term representing the deviation of the computed final state from the given final state condition is

added here to guarantee that the simulation result satisfies the final state condition, and the second term represents the fuel consumption during the time interval. Here R is a normalizing parameter for the torque control. Without loss of generality, R is assumed to be 1 for the following discussion. The corresponding Hamiltonian is

$$H(\mathbf{x}_r, \boldsymbol{\lambda}, \mathbf{u}) = \boldsymbol{\lambda}^T \mathbf{f}(\mathbf{x}_r) + \sum_{i=1}^3 \boldsymbol{\lambda}^T \mathbf{g}_i(\mathbf{x}_r) u_i + \gamma(|u_1(t)| + |u_2(t)| + |u_3(t)|) \quad (11)$$

Clearly the adjoint equations are still in the form of (9).

With the formulation we can apply SS algorithm to solve the fuel consumption optimal control problem in the time interval $[0, t_1]$. The numerical simulation starts with an initial guess of the optimal controller, $\mathbf{u}(t)$, and then iteratively finds an extremal $(\mathbf{x}_r(t), \boldsymbol{\lambda}(t), \mathbf{u}(t))$ that satisfies the Pontryagin's Maximum Principle and the final state condition. The iterations will stop at the i -th time if both conditions

$$\frac{1}{2} \|\mathbf{x}_r^i(t_1) - \mathbf{x}_{r,f}\|^2 < \varepsilon_1 \quad (12)$$

and

$$\int_0^{t_1} \|\mathbf{u}^i(t) - \mathbf{u}^{i-1}(t)\|^2 dt < \varepsilon_2 \quad (13)$$

are satisfied. In the above conditions ε_1 and ε_2 are given small positive numbers as thresholds to judge the convergence of the iteration. Due to the space limitations, the detailed procedure of Sakawa-Shindo algorithm will not be presented here.

In the planar example, the angular velocity of the target is assumed to be $\Omega = 0.1 \text{ rad/s}$. The initial condition is taken as

$$\mathbf{x}_{r,0} = (10 \ 10 \ \pi/2 \ 1 \ 1 \ 1)^T$$

and the desired final state is $\mathbf{x}_{r,f} = (1 \ 0 \ 0 \ 0 \ 0 \ 0)^T$. The normalized control input are assumed to be $|u_1| = |u_2| = 1 \text{ N/kg}$ and $|u_3| = 1 \text{ N/(kg}\cdot\text{m}^2)$. All computational parameters are taken as the same values as in [10], in which $\sigma = 1$, $\gamma = 0.005$, $\varepsilon_1 = 0.001$, and $\varepsilon_2 = 0.001$. The Heun method is applied for solving differential equations (7) and (9). $[0, t_1]$ is divided into 200 segments for numerical integration.

The accompanying plots show results for the value of $t_1 = 8.14\text{s}$. The corresponding value of the cost function J in equation (10) is 0.0788. Fig.4 shows the optimal trajectories starting from $\mathbf{x}_{r,0}$ and ending at $\mathbf{x}_{r,f}$. The chaser's corresponding optimal moving path in the target's body fixed frame X_1Y_1 is shown in Fig. 5. Fig. 6 shows the profiles of the optimal controls $u_1(t)$ and $u_2(t)$. The accuracy of the simulation results depends on the convergence property of the algorithm, the values of those control parameters (including the values of thresholds ε_1 and ε_2), the numerical integration method, and the number of division of the time interval $[0, t_1]$.

It should be remarked that in applying the SS algorithm in the time interval $[0, t_1]$ to find a corresponding fuel consumption optimal control candidate, the leverage one has in choosing t_1 allows one to optimize, to a certain extent, the time of the trajectory as well. That is, the algorithm can be iteratively implemented for smaller values of t_1 : Once the fuel optimal control candidate for a chosen time interval $[0, t_1]$ is found, the value of t_1 can then be decreased and the computing process repeated until it converges to a minimal value. Indeed for the above plots, the initial choice for t_1 was taken as 12s. Over repeated runs of the numerical optimization process, t_1 was successively decreased till a smallest value of $t_1 = 8.14\text{s}$ was obtained. The plots even exhibit some characteristics of 'bang-bang' type of control as we have expected from the theoretical analysis of the time-optimal control problem. Strictly speaking, this result cannot be regarded as a candidate of time-optimal control, for it is obtained from a numerical algorithm for searching fuel consumption optimal control. But it can be regarded as a reasonable candidate for the combined time and fuel consumption optimal control problem.

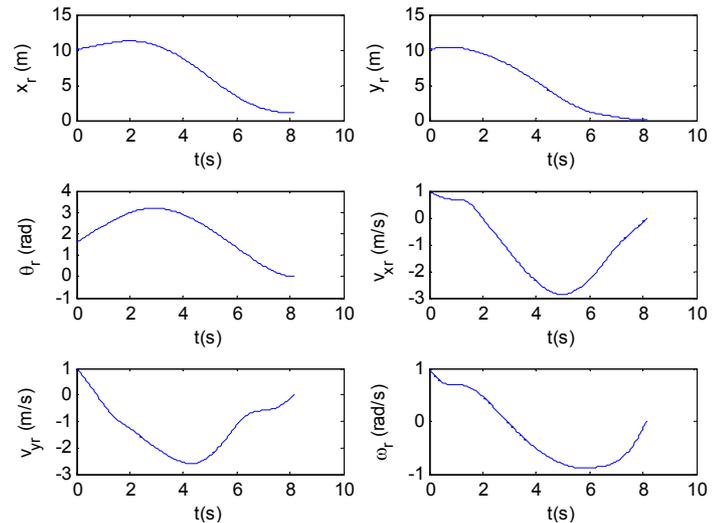


Fig. 4 Optimal trajectories

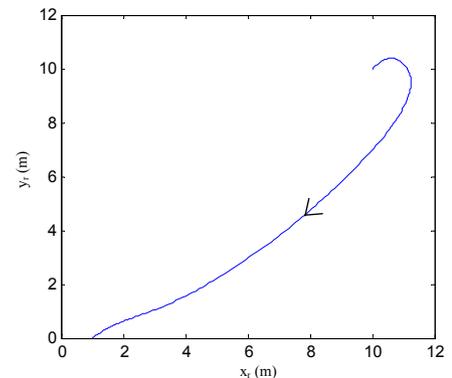


Fig. 5 Optimal trajectory of the Chaser in X_1 - Y_1 plane

Finally, as to the computational cost for running the simulations, we compared, with a fixed t_1 , the computational cost of running the SS algorithm for a particular fuel-optimal

control problem with those of several other algorithms presented in [18]. The SS algorithm proved efficient in the sense that convergence to the final solution was obtained with a not large number of iterations. Moreover, since at each step the Pontryagin's Maximum Principle is satisfied, the SS algorithm is based on a more solid theoretical base than many 'direct methods' based on nonlinear programming optimization methods (see [19]). In our planar example, the simulation for a fixed t_1 case does not cost much computational time though it is not real-time yet. To solve our time-optimal control problem, we need to reduce t_1 step by step until it converges to minimum. This is therefore an 'iteration of iteration' process and costs more time for computation. However, with a reasonable initial guess of t_1 (such as taking the initial t_1 as 12s in our example), the computational cost is still not high. A typical case run using Matlab 6.5 on a 1-GHz Pentium IV PC takes several minutes. The computational efficiency is not our major focus at this stage because the optimal control problem is used only for optimal trajectory planning (feedforward control).

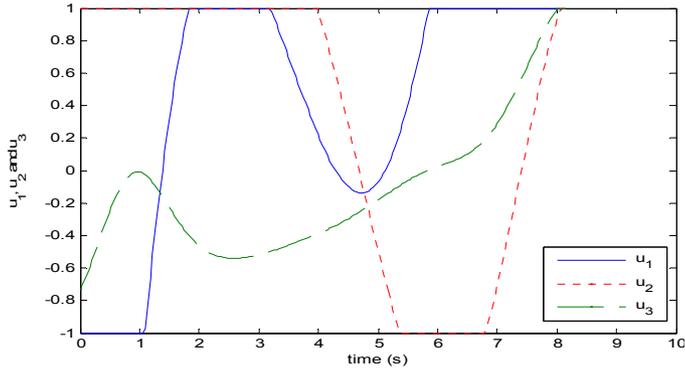


Fig. 6 Optimal control force and torque profiles

V. CONCLUSIONS

In this paper, the optimal closed-range rendezvous problem for a servicing spacecraft to approach a tumbling satellite for capture is formulated and analyzed. Pontryagin's Maximum Principle is applied to investigate the optimal control candidate $\mathbf{u}^*(t)$ that minimizes the cost function along the trajectory of the chaser as it moves from its initial state to a given final state. When the control goal is achieved, the chaser is at a fixed distance and has zero relative motion with respect to the rotating target. The control inputs are normalized and thus the analysis results will be applicable to spacecraft of different mass and inertia values. A planar example is presented to demonstrate the proposed optimal control method. The theoretical analysis shows that for time-optimal control in the example at least one control input must be bang-bang during the time interval. Numerical simulation results based on Sakawa-Shindo algorithm are then performed for the fuel consumption optimal control with the same example. In the future, we will extend these techniques and perform computations using a more realistic example of a three-

dimensional rendezvous problem.

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